612-MATHEMATICS
(FINAL)

1. The sequence $\{x_n\}$ where $x_n = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n-1}$ is

(A) increasing but not bounded
(B) decreasing and bounded
(C) increasing and bounded
(D) decreasing but not bounded

2. If A and B are sets such that $O(A) = 5$ and $O(B) = 3$, then the number of binary relations from A to B is

(A) 2^8
(B) 2^9
(C) 2^{15}
(D) 2^{24}

3. Let $R = \{(a,a),(b,c),(a,b)\}$ be a relation on the set $\{a,b,c\}$. The minimum number of elements that should be added to R so that it becomes antisymmetric are

(A) 2
(B) 3
(C) 1
(D) 0

4. If C is a circle $|z| = 1$, then $\int_C \overline{z}\, dz$ is

(A) πi
(B) $2\pi i$
(C) 0
(D) None of the above

5. The n^{th} roots of unity under multiplication form a

(A) monoid
(B) groupoid
(C) semigroup
(D) abelian group

6. The Legendre equation is given by

(A) $x^2 y'' + xy' + \left(x^2 - n^2\right) y = 0$

(B) $\left(1 - x^2\right) y'' + 2xy' - n\left(n+1\right) y = 0$

(C) $\left(1 - x^2\right) y'' - 2xy' + n\left(n+1\right) y = 0$

(D) $\left(1 - x^2\right) y'' - xy' + \left(x^2 - n^2\right) y = 0$

7. The set of rational numbers of the form $\frac{m}{2^n}$ (*m*, *n* are integers) is a group under

(A) addition
(B) subtraction
(C) multiplication
(D) division

8. The generators of the group $G = \left\{a, a^2, a^3, a^4 = e\right\}$ are

(A) a only

(B) a and a^2

(C) a and a^3

(D) a and a^4

9. The solution of $\frac{dy}{dx} = y^2$, $y(0) = 1$, exists for all

(A) $x \in (-\infty, 1)$
(B) $x \in [0, a]$ where $a > 1$
(C) $x \in (-\infty, \infty)$
(D) $x \in [1, a]$ where $a > 1$

10. The value $\int_C \bar{F}.d\bar{r}$, where $\bar{F} = x^2 y^2 \vec{i} + y\vec{j}$ and C is $y^2 = 4x$ from (0, 0) to (4, 4), is

(A) 64
(B) -128
(C) 128
(D) 264

11. The directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q has the coordinates (5, 0, 4) is

(A) $\frac{28}{\sqrt{15}}$

(B) $\sqrt{\frac{13}{21}}$

(C) $\frac{4}{\sqrt{15}}$

(D) $\frac{28}{\sqrt{21}}$

12. If $\{\alpha, \beta\}$ is an orthonormal set, then the distance between α and β is

(A) $\sqrt{3}$
(B) 0
(C) 3
(D) $\sqrt{2}$

13. The function $f(z) = z^2$ is differentiable

(A) nowhere
(B) everywhere
(C) only at 0
(D) only in $|z| < 1$

14. The equation of the plane passing through the points (−1, 1, 1) and (1, −1, 1) and perpendicular to the plane $x + 2y + 2z = 7$ is

(A) $4x + 2y - 3z + 5=0$
(B) $4x + 4y - 6z + 5=0$
(C) $2x + 2y - 3z + 3=0$
(D) $2x + 2y - 6z + 3=0$

15. The total number of subgroups of Z contained in $20Z$ is

(A) 6
(B) 2
(C) infinite
(D) 18

16. Which of the following statement is true?

(A) $(Q, +)$ is a cyclic group
(B) Every abelian group is cyclic
(C) Every group of order < 4 is cyclic
(D) Every element of a cyclic group generates the group

17. The equation of a right circular cone with vertex at origin 0, axis the x-axis and semi-vertical angle α is

(A) $x^2 + y^2 = x^2 \operatorname{sech}^2\alpha$

(B) $y^2 + z^2 = x^2 \tanh^2 \alpha$

(C) $y^2 + z^2 = x^2 \tan^2 \alpha$

(D) $x^2 + y^2 = x^2 \tanh^2 \alpha$

18. Let R be the ring of all real valued functions defined on R, under pointwise addition and multiplication. Which of the following subsets of R is not a subring?

(A) Set of all continuous functions
(B) Set of all polynomial functions
(C) Set of all functions which are zero at finitely many points together with the zero function
(D) Set of all functions which are zero at countable number of points

19. Let $T : R^2 \rightarrow R^2$ be a linear transformation such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$. Then T maps the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) into a

(A) rectangle
(B) trapezium
(C) square
(D) parallelogram

20. The locus of the complex number satisfying $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ is a

(A) straight line
(B) circle
(C) parabola
(D) hyperbola

21. If z_1, z_2 are two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then it is necessary that

(A) $z_1 = z_2$

(B) $z_2 = 0$

(C) $z_1 = \lambda z_2$ for all real number λ

(D) $z_1 z_2 = 0$ or $z_1 = \lambda z_2$ for some real number λ

22. Consider the function $f(x) = \begin{cases} x - \sin x, & \text{if } x \text{ is rational} \\ 0 & \text{,if } x \text{ is irrational} \end{cases}$. Then

(A) $f(x)$ is everywhere discontinuous
(B) $f(x)$ is continuous at one point
(C) $f(x)$ is continuous more than one point but at countable points
(D) $f(x)$ is continuous at exactly two points

23. The function $f(x) = |x| + |x-1|$, $x \in R$ is

(A) continuous but not differentiable at $x = 0$ and $x = 1$
(B) discontinuous at $x = 0$ and $x = 1$
(C) discontinuous at $x = 0$ and not differentiable at $x = 1$
(D) differentiable everywhere

24. The plane $x + 2y - z = 4$ and the sphere $x^2 + y^2 + z^2 + x + z - 2 = 0$

(A) do not meet each other
(B) intersect at only one point
(C) intersect along a circle of unit radius
(D) intersect along the great circle

25. The number of elements of order 11 in a group of order 33 are

(A) 0
(B) 10
(C) 20
(D) 30

26. The value of $div\left(r^n\overline{r}\right)$ is

(A) $(n+3)r$

(B) $(n+3)r^{-2n}$

(C) $(n+3)r^n$

(D) $nr^{-(n+3)}$

27. Green's theorem applied to $\int_C(\cos x\sin y-xy)dx+\sin x\cos y dy,$ where C is the circle $x^2+y^2=1,$ yields

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 0

(D) $\frac{2}{3}$

28. The units of Z_6 are

(A) $1, -1$
(B) 1, 2, 3, 5
(C) 1, 5
(D) 1, 2, 3, 4

29. The only function among the following that is analytic is

(A) $f(z)=\mathrm{Re}(z)$
(B) $f(z)=\mathrm{Im}(z)$
(C) $f(z)=\overline{z}$
(D) $f(z)=\sin z$

30. The series $g(z)=\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n^2\pi z$ is

(A) not absolutely convergent
(B) uniformly convergent
(C) convergent but not uniformly convergent
(D) not uniformly convergent

31. Let A be a real symmetric matrix. Then which of the following is true?

(A) A does not have 0 as an eigen value

(B) A has at least one positive eigen value

(C) If A^{-1} exists, then A^{-1} is real and symmetric

(D) All eigen values of A are complex numbers

32. The largest possible order of elements in S_7 is

(A) 7
(B) 12
(C) 8
(D) 6

33. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}-\frac{d^2y}{dx^2}=0$ is

(A) $\{1, x, e^{x}, e^{-x}\}$

(B) $\{1, x, e^{-x}, xe^{-x}\}$

(C) $\{1, x, e^{x}, xe^{x}\}$

(D) $\{1, x, e^{x}, xe^{-x}\}$

34. The least natural number $\boldsymbol{a}$ for which $x+ax^{-2}>2$ for all $x\in R^{+}$

(A) 1
(B) 2
(C) 5
(D) 9

35. The orthogonal trajectories of the family $x^2 - y^2 = C_1$ are given by

(A) $x^2 + y^2 = C_2$

(B) $xy = C_2$

(C) $y = C_2$

(D) $x = C_2$

36. Let C be the positively oriented unit circle $|z| = 1$. Then the integral $\int_C \frac{e^{i\pi z}\sin(\pi z)z^5}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)}dz$ is equivalent to

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{4}$

(C) $-\frac{\pi}{8}$

(D) $\frac{\pi}{8}$

37. Any three vectors $\{x_1, x_2, x_3\}$ in $\mathbb{R}^2$

(A) are linearly independent

(B) are linearly dependent

(C) form a basis for $\mathbb{R}^2$

(D) generate a non-trivial subspace of $\mathbb{R}^2$

38. The number abelian groups of order 8 is

(A) 2
(B) 3
(C) 1
(D) 4

39. The integral surface of the partial differential equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ satisfying the condition $u(1, y) = y$ is given by

(A) $u(x, y) = \frac{y}{x}$

(B) $u(x, y) = \frac{2y}{x+1}$

(C) $u(x, y) = \frac{y}{2-x}$

(D) $u(x, y) = y + x - 1$

40. The probability that a single toss of a die will result in a number less than 4 if the toss resulted in an odd number is

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{1}{2}$

41. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map which maps each point in $\mathbb{R}^2$ to its reflection on the *x*-axis. Then the determinant and trace of *T* are given by

(A) determinant = 1, trace = 0
(B) determinant $= -1$, trace = 1
(C) determinant $= -1$, trace = 0
(D) determinant = 2, trace = 1

42. The radius of convergence of $\sum \frac{n^2}{2^n} x^n$ is

(A) 2

(B) $\frac{1}{2}$

(C) 1

(D) ∞

43. The inverse Laplace transform of $\dfrac{s}{s^4+4a^4}$ is

(A) $-\dfrac{1}{2a}\sin at \sinh at$

(B) $-\dfrac{1}{2a^2}\sin at \sinh at$

(C) $\dfrac{1}{2a}\sin at \sinh at$

(D) $\dfrac{1}{2a^2}\sin at \sinh at$

44. The number of generators of a cyclic group of order 8 is

(A) 2
(B) 4
(C) 6
(D) 8

45. Let $M(n,\mathbb{R})$ be the vector space of $n\times n$ matrices with real entries. Let U be the subset of $M(n,\mathbb{R})$ consisting $\left\{(a_{ij}) \mid a_{11}+a_{22}+...+a_{nn}=0\right\}$. Then the dimension of the subspace U is

(A) $\dfrac{n(n+1)}{2}$

(B) n^2

(C) n^2-1

(D) $\dfrac{n(n-1)}{2}$

46. The general integral of $yzp+zxq=xy$ is

(A) $f(x+y, y+z)=0$

(B) $f(x^2+y^2, x^2+z^2)=0$

(C) $f(x^2-y^2, x^2-z^2)=0$

(D) $f(x+y+z, x)=0$

47. Let $f:[-1,1]\to \mathbb{R}$ be defined by $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$, where $\mathbb{R}$ is the set of all real numbers. Then

(A) f satisfies the condition of Rolles theorem on [1, 1]
(B) f satisfies the condition of Lagranges mean value theorem on [1, 1]
(C) f satisfies the condition of Rolles theorem on [0, 1]
(D) f satisfies the condition of Lagranges mean value theorem on [0, 1]

48. The expectation of a discrete random variable *X*, whose probability function is given by $f(x)=\left(\frac{1}{2}\right)^{x}$, $x=1,2,3...,$ is

(A) 1

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{1}{4}$

49. If $G\neq\{e\}$ is a group having no proper subgroup, then *G* is a

(A) cyclic group of prime order
(B) cyclic group of even order
(C) cyclic group of odd order
(D) group of even order

50. If *D* is the region defined by $1\leq x^2+y^2+z^2\leq 2$ and $z\geq 0,$ then $\iiint_D e^{\left(x^2+y^2+z^2\right)^{3/2}}\,dx\,dy\,dz$ is equal to

(A) $\frac{2\pi}{3}\left(e^{\sqrt{8}}-e\right)$

(B) $2\pi\left(e^{\sqrt{8}}-e\right)$

(C) $\frac{\pi}{3}\left(e^{\sqrt{8}}-e\right)$

(D) $\frac{\pi}{2}\left(e^{\sqrt{8}}-e\right)$

51. The values of a and b if the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$ is

(A) $a = -5/2, \ b = -1$

(B) $a = 5/2, b = -1$

(C) $a = -5/2, b = 1$

(D) $a = 5/2, b = 1$

52. The interior of the set $\left\{ r \in \square : 0 < r < \sqrt{2} \right\}$ is

(A) $\square$
(B) $\square$
(C) ϕ
(D) $\square - \{0\}$

53. The general solution of $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ is of the form

(A) $u = f(x+iy) + g(x-iy)$
(B) $u = f(x+y) + g(x-y)$
(C) $u = cf(x-iy)$
(D) $u = g(x+iy)$

54. The general integral of the partial differential equation $(y+zx)z_x - (x+yz)z_y = x^2 - y^2$ is

(A) $F\left(x^2 + y^2 + z^2, xy + z\right) = 0$

(B) $F\left(x^2 + y^2 - z^2, xy + z\right) = 0$

(C) $F\left(x^2 - y^2 - z^2, xy + z\right) = 0$

(D) $F\left(x^2 + y^2 + z^2, xy - z\right) = 0$

55. Two cards are drawn from a well-shuffled ordinary deck of 52 cards. The probability that they are both aces if the first card is replaced is

(A) $\frac{1}{221}$

(B) $\frac{1}{169}$

(C) $\frac{1}{122}$

(D) $\frac{1}{196}$

56. If $f(x)$ is a polynomial of degree n in x, then n^{th} difference of this polynomial is

(A) a constant
(B) n
(C) zero
(D) 1

57. Consider the following:

(i) $n^2 + n$ is divisible by 2

(ii) $n^3 - n$ is divisible by 3

(iii) $n^5 - 5n^3 + 4n$ is divisible by 5

Then

(A) Only (i) is true
(B) Only (i) and (ii) are true
(C) Only (i) and (iii) are true
(D) All the three are true

58. Consider the following statements.

(i) The product of four consecutive integers is divisible by 24
(ii) The product of five consecutive positive integers is a perfect square
(iii) There exist infinitely many positive integers n such that $\dfrac{n}{2^n+1}$
(iv) $\dfrac{6}{n(n+1)(2n+1)}$ for each positive integer n

Then

(A) (i) is true
(B) (ii) is not true
(C) (iii) is true
(D) (iv) is true

59. Let $A=\left(a_{ij}\right)$ be a 2×2 lower triangular matrix with diagonal entries $a_{11}=1$ and $a_{22}=3$. If $A^{-1}=\left(b_{ij}\right)$ then

(A) $b_{11}=1,\ b_{22}=1$
(B) $b_{11}=1,\ b_{22}=\dfrac{1}{3}$
(C) $b_{11}=3,\ b_{22}=2$
(D) $b_{11}=3,\ b_{22}=1$

60. Consider $A=\left\{(x,y):(x+1)^2+y^2\le 1\right\}\cup\left\{(x,y): y=x\sin\dfrac{1}{x}, x>0\right\}\subseteq\mathbb{R}^2$. Then

(A) A is compact
(B) A is connected
(C) A is bounded
(D) A is not connected

61. The differential equation whose linearly independent solutions are $\cos 2x$, $\sin 2x$ and e^{-x} is

(A) $\left(D^3 + D^2 + 4D + 4\right) y = 0$

(B) $\left(D^3 - D^2 + 4D - 4\right) y = 0$

(C) $\left(D^3 + D^2 - 4D - 4\right) y = 0$

(D) $\left(D^3 - D^2 - 4D + 4\right) y = 0$

62. The smallest integer $a > 2$ such that $2 \mid a,\ 3 \mid a+1,\ 4 \mid a+2,\ 5 \mid a+3,\ 6 \mid a+4$ ($a \mid b$ mean a divides b) is

(A) 24
(B) 12
(C) 62
(D) 74

63. In a group of 100 people, each one knows at least 67 other people. Then the minimum number of people who are mutual friends is

(A) 0
(B) 2
(C) 3
(D) 4

64. A solution of the equation $\log_4\left(2x^2 + x + 1\right) - \log_2(2x-1) = 1$ is

(A) $-\frac{3}{14}$

(B) -1

(C) 1

(D) $\frac{6}{14}$

65. The second moment about the origin of the exponential distribution
$f(x) = \frac{1}{c} e^{-x/c}, \ 0 \le x \le \infty, \ c > 0,$ is

(A) $2c^2$

(B) c^3

(C) c^2

(D) $2c^3$

66. Suppose that B is a square matrix such that $B^2 = B$ and that $\lambda = 1$ is not an eigenvalue of B. Then B is

(A) a zero matrix
(B) a symmetric matrix
(C) a diagonal matrix
(D) any arbitrary non zero matrix

67. The set of all points of discontinuity of the function $f : [-1,1] \to \mathbb{R}$ defined by
$f(x) = \begin{cases} x, \text{ is if } x \text{ is irrational} \\ 0, \text{ is if } x \text{ is rational} \end{cases}$ is

(A) $[-1,1]$
(B) $(-1,1)$
(C) $[-1,1]-\{0\}$
(D) ϕ

68. Consider the statements.

(i) In a tree, every edge is a bridge
(ii) In a tree, every non pendant point is a cut point
(iii) Any connected graph (p,q) with $p+1=q$ is a tree
(iv) Every tree is a bipartite graph

Then

(A) (i) is not true
(B) (ii) is not true
(C) (iii) is not true
(D) (iv) is not true

69. The number of non-zero homomorphisms from $\mathbb{Z}_8$ to $\mathbb{Z}_{10}$ is

(A) 8
(B) 10
(C) 2
(D) 4

70. The bilinear transformation has ∞ and one finite point as fixed points, when

(A) $c \neq 0;\ (d-a)^2 + 4bc \neq 0$

(B) $c \neq 0;\ (d-a)^2 + 4bc = 0$

(C) $c = 0;\ a \neq d$

(D) $c = 0;\ a = d$

71. The solution of $\dfrac{dy}{dx} = \dfrac{y-x+1}{y-x+5}$ is

(A) $(x-y)^3 + 10y - 5x = C$

(B) $(y-x)^2 + 10y - 2x = C$

(C) $(x+y)^2 - 5y + 10x = C$

(D) $(x+y)^3 - 10y - 2x = C$

72. The sequence $\{1 + (-1)^n\}$ has

(A) exactly one constant subsequence
(B) exactly two constant subsequences
(C) exactly three constant subsequences
(D) exactly four constant subsequences

73. Which amongst the following expressions is **NOT** true?

(A) $\lim\limits_{n\to\infty} \dfrac{3n+2}{n+1} = 1$

(B) $\lim\limits_{n\to\infty} n\left(\dfrac{3n+7}{3n+3} - 1\right) = \dfrac{4}{3}$

(C) $\lim\limits_{n\to\infty} \dfrac{3+2\sqrt{n}}{\sqrt{n}} = 2$

(D) $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$

74. Which amongst the following statements is **NOT** true?

(A) A sequence cannot converge to move than one limit
(B) Every convergent sequence is bounded
(C) Every bounded sequence is convergent
(D) Limit of a sequence is unique

75. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence converges to 0 and $\{b_n\}_{n=1}^{\infty}$ be a bounded sequence. Then $\{a_n b_n\}_{n=1}^{\infty}$

(A) converges to one
(B) converges to zero
(C) is a divergent sequence
(D) converges to two

76. Let $\sum a_n$ be a convergent series of positive terms and let $\sum b_n$ be a divergent series of positive terms. Then

(A) the sequence $\langle a_n \rangle$ is convergent and $\langle b_n \rangle$ is not convergent
(B) the sequence $\langle a_n \rangle$ converges to 0
(C) the sequence $\langle b_n \rangle$ does not converges to 0
(D) the sequence $\langle b_n \rangle$ diverges to ∞

77. If we expand $\sin x$ by Taylor's series about $\frac{\pi}{2}$, then a_2, a_7, a_4, a_3 are

(A) $-\frac{1}{2}, 0, \frac{1}{24}, 0$
(B) $-\frac{1}{2}, 0, -\frac{1}{24}, 0$
(C) $\frac{1}{2}, 0, \frac{1}{24}, 0$
(D) $0, -\frac{1}{2}, 0, \frac{1}{24}$

78. If $f(x)$ and $g(x)$ are two functions such that $f'(x) = g'(x)$ on an interval *I*, then the difference function $f - g$ is

(A) the zero function on *I*
(B) a constant function on *I*
(C) a polynomial of degree one on *I*
(D) a polynomial of degree two on *I*

79. The function $f(x) = |x| + 3$ is

(A) continuous as well as differentiable on *R*
(B) continuous on *R* but not differentiable anywhere on *R*
(C) not continuous on *R*
(D) continuous on *R* and differentiable at all points on *R* except zero

80. Let $S = \{(1, i, 0), (2i, 1, 1), (0, 1 + i, 1 - i)\}$ be a subset of the complex vector space C^3 and $T = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ be the subset of the real vector space R^3. Which of the following is correct?

(A) *S* and *T* are both basis
(B) *S* is a basis but *T* is not a basis
(C) *S* is not a basis but *T* is a basis
(D) Neither *S* nor *T* is a basis

81. If $X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then the rank of $X^T X$,

(where X^T denotes the transpose of X), is

(A) 0
(B) 2
(C) 3
(D) 4

82. The system of equations $x - y + 3z = 4$, $x + z = 2$ and $x + y - z = 0$ has

(A) unique solution
(B) finitely many solutions
(C) infinitely many solutions
(D) no solution

83. If A be a non-zero square matrix of order n, then

(A) the matrix $A+A'$ is anti-symmetric, but the matrix $A-A'$ is symmetric

(B) the matrix $A+A'$ is symmetric, but the matrix $A-A'$ is anti-symmetric

(C) both $A+A'$ and $A-A'$ matrices are symmetric

(D) both $A+A'$ and $A-A'$ matrices are anti-symmetric

84. Let $A=\begin{pmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{pmatrix}$. Then

(A) A is diagonalizable but not A^2

(B) A^2 is diagonalizable but not A

(C) both A and A^2 are diagonalizable

(D) neither A nor A^2 is diagonalizable

85. Let T_1 and T_2 be two linear operators on R^3 defined by $T_1(x,y,z)=(x,x+y,x-y-z)$ and $T_2(x,y,z)=(x+2z,y-z,x+y+z)$. Then

(A) T_1 is invertible but not T_2

(B) T_2 is invertible but not T_1

(C) both T_1 and T_2 are invertible

(D) neither T_1 nor T_2 is invertible

86. The complex numbers $z_1=1+2i$, $z_2=4-2i$ and $z_3=1-6i$ form the vertices of

(A) a right angled triangle
(B) an isosceles triangle
(C) an equilateral triangle
(D) a scalene triangle

87. The radius of convergence of the power series $\sum_{n=0}^{\infty}(n+2i)^n z^n$ is

(A) 0
(B) 1
(C) ∞
(D) 2

88. The center of convergence of $\sum_{n=0}^{\infty}(n+2i)^n z^n$ is

(A) 0
(B) 1
(C) 2
(D) 3

89. The function $f(z)=\log\left(z^2+z-2\right)$ has branch poles at

(A) $z=1,\ z=-1$
(B) $z=2,\ z=-2$
(C) $z=1,\ z=-2$
(D) $z=3,\ z=-2$

90. The bilinear transformation which maps the points $z_1=2,\ z_2=i$ and $z_3=-2$ into the points $w_1=1,\ w_2=i$ and $w_3=-1$ respectively, is

(A) $w=\dfrac{3z+2i}{iz+6}$
(B) $w=0$
(C) $w=iz$
(D) $w=\dfrac{z^2}{2}$

91. Numbers between 5000 and 10000 are formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit not appearing more than once in each number. The number of such numbers is

(A) 1080
(B) 1680
(C) 336
(D) 1008

92. If $\gcd(a,b)=1$, then $\gcd(a+b, a^2-ab+b^2)$ is

(A) 0 or 1
(B) 1 or 2
(C) 1 or 3
(D) 2 or 3

93. Number of the solutions of the congruence $15x \equiv 6 \pmod{21}$ is

(A) 1
(B) 2
(C) 3
(D) 4

94. If $ac \equiv bc \pmod{m}$ and $d=(m,c)$, then

(A) $a \equiv b\left(\bmod \dfrac{m}{d}\right)$

(B) $a \equiv c\left(\bmod \dfrac{m}{d}\right)$

(C) $a \equiv m\left(\bmod \dfrac{b}{d}\right)$

(D) $a \equiv b (\bmod c)$

95. Let $G=\{(0,1,2,3,4),+_5\}$. Then the order of 2 in G is

(A) 1
(B) 2
(C) 4
(D) 5

96. Given the permutation $c=(1\quad 2\quad 3\quad 4\quad 5\quad 6\quad 7)$. Then c^3 is

(A) $(1\quad 3\quad 5\quad 7\quad 2\quad 4)$

(B) $(1\quad 4\quad 7\quad 3\quad 6\quad 2\quad 5)$

(C) $(1\quad 7\quad 6\quad 5\quad 4\quad 3\quad 2)$

(D) $(1\quad 4\quad 7\quad 6\quad 3\quad 5\quad 2)$

97. Suppose G is a group and N is a normal subgroup of G. Let $F : G \to \frac{G}{N}$ defined by $F(x) = Nx \ \forall \ x \in G$. Then

(A) F is a homomorphism of G into $\frac{G}{N}$ with $\ker F \neq N$

(B) F is a homomorphism of G onto $\frac{G}{N}$ with $\ker F \neq N$

(C) F is a homomorphism of G into $\frac{G}{N}$ with $\ker F = N$

(D) F is a homomorphism of G onto $\frac{G}{N}$ with $\ker F = N$

98. Which of the following is a compact subspace of R?

(A) $\{0\} \cup \{n \mid n \in z_+\}$

(B) The subspace $X = \{0\} \cup \left\{\frac{1}{n} \mid n \in z_+\right\}$ of R

(C) [0, 1)

(D) (0, 1)

99. The real part of the complex number $(1+i)^n$ is

(A) $2^{\frac{n}{2}} \cos \frac{n\pi}{4}$

(B) $2^n \cos \frac{n\pi}{2}$

(C) $2^{\frac{n}{2}} \cos n\pi$

(D) $2^{-n} \cos \frac{n\pi}{2}$

100. If $|z| = |z-1|$, then equal to

(A) $\operatorname{Re}(z) = 1$

(B) $\operatorname{Re}(z) = \frac{1}{2}$

(C) $\operatorname{Im}(z) = 1$

(D) $\operatorname{Im}(z) = \frac{1}{2}$

101. The value of the integral $\oint_C \frac{e^z}{z-2}\, dz$ where $C : |z| = 3$ is

(A) $2\pi i e^2$

(B) $2\pi i e$

(C) e^2

(D) $2\pi i$

102. The unit digit of 2^{100} is

(A) 2
(B) 4
(C) 6
(D) 8

103. The number of generators in a cyclic group of order 10 is

(A) 3
(B) 1
(C) 2
(D) 4

104. If α and β are the eigenvalues of $\begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$, then the matrix whose eigenvalues are α^3 and β^3 is

(A) $\begin{bmatrix} 38 & -50 \\ -50 & 138 \end{bmatrix}$

(B) $\begin{bmatrix} -50 & 38 \\ 138 & -50 \end{bmatrix}$

(C) $\begin{bmatrix} -50 & -50 \\ 38 & 138 \end{bmatrix}$

(D) $\begin{bmatrix} -50 & -50 \\ 138 & 38 \end{bmatrix}$

105. Given that the equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have an infinite number of solutions. Then

(A) $\lambda = 10, \mu = 3$
(B) $\lambda = 5, \mu = 5$
(C) $\lambda = 3, \mu = 10$
(D) $\lambda = 2, \mu = 10$

106. $\lim\limits_{x \to 0} \dfrac{\sinh x - \sin x}{x^3}$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{5}$

107. If $\dfrac{\tan\theta}{\theta} = \dfrac{2524}{2523}$, then θ is equal to

(A) $\dfrac{1}{28}$ radians

(B) $\dfrac{1}{29}$ radians

(C) $\dfrac{1}{30}$ radians

(D) $\dfrac{1}{25}$ radians

108. $\sinh^{-1}\sqrt{x^2-1} =$

(A) $\sinh^{-1} x$

(B) $\tanh^{-1} x$

(C) $\cosh^{-1} x$

(D) $\cot^{-1} x$

109. An equation of the form $z\bar{z} + b\bar{z} + \bar{b}z + c = 0$ (c is real) represents

(A) a circle
(B) an ellipse
(C) a parabola
(D) a hyperbola

110. The transformation $w = \cos z$ makes the line $x = a$ in the z-plane corresponds to

(A) a hyperbola
(B) a parabola
(C) an ellipse
(D) a circle

111. The residue of $\dfrac{z^2}{z^2+a^2}$ at $z=ia$ is

(A) ia

(B) $2ia$

(C) $\dfrac{1}{2}ia$

(D) $3ia$

112. The standard deviation of a symmetrical distribution is 3. The fourth moment about mean (assuming that the distribution is mesokurtic) is

(A) 240
(B) 241
(C) 242
(D) 243

113. The coefficients of correlation between two variables x and y is 0.8. The covariance between x and y is 18 and the standard deviation of x is 3. The standard deviation of y is

(A) 7
(B) 8
(C) 7.5
(D) 8.5

114. If the two regression coefficients are 0.8 and 0.2, then the value of correlation coefficient is

(A) $r = 0.2$
(B) $r = 0.3$
(C) $r = 0.1$
(D) $r = 0.4$

115. If $\overline{a}$ is constant vector and $\overline{r}$ is the position vector of any point, then $\operatorname{div}\left(\overline{a}\times\overline{r}\right)$ is

(A) 0
(B) 1
(C) 2
(D) 3

116. If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $r = |\overline{r}|$, then $r^n \overline{r}$ is solenoidal, if

(A) $n = -2$
(B) $n = -3$
(C) $n = -1$
(D) $n = 1$

117. The image of $x + y = 2$ under the transformation $w = z^2$ is

(A) a hyperbola
(B) a circle
(C) an ellipse
(D) a parabola

118. The series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots$ is

(A) divergent
(B) convergent
(C) oscillating
(D) converges to 10

119. If $c_0, c_1, c_2, \ldots, c_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then $c_1 + 2c_2 + 3c_3 + \ldots nc_n =$

(A) $n.2^n$
(B) $n.2^{n+1}$
(C) $n.2^{n-1}$
(D) 2^n

120. Sum the series to n terms: $\frac{3}{1.4}+\frac{5}{4.9}+\frac{7}{9.16}+\frac{9}{16.25}+\ldots$

(A) $1+\frac{1}{(n+1)^2}$

(B) $1-\frac{1}{(n+1)^2}$

(C) $1-\frac{1}{(n-1)^2}$

(D) $1+\frac{1}{(n-1)^2}$

121. A particle moves along a straight line according to the law $s = t^3 - 6t^2 + 9t^2 + 3$. The velocity, at the instant where its acceleration is zero, is

(A) –3 unit/sec
(B) –2 unit/sec
(C) 3 unit/sec
(D) 2 unit/sec

122. The value of (to five places of decimals) cube root of 1003 is

(A) 10.00333
(B) 10.00666
(C) 10.00444
(D) 10.00999

123. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ is a

(A) perfect square
(B) quadratic
(C) biquadratic
(D) perfect cube

124. The value of the point c of the Mean value theorem for the function $f(x)=1-x^2$ in $0 \le x \le 2$ is

(A) $c=2$
(B) $c=3$
(C) $c=0$
(D) $c=1$

125. The area of the region bounded by the parabola $y = 2 - x^2$ and the line $y = -x$ is

(A) $\frac{9}{4}$

(B) $\frac{3}{4}$

(C) $\frac{9}{2}$

(D) $\frac{5}{2}$

126. $\lim_{x \to 0} \frac{4-\sqrt{16+x}}{x}$ is equal to

(A) $\frac{1}{8}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $-\frac{1}{4}$

127. Ten percent of the tools produced in a certain manufacturing process turn out to be defective. The probability that in a sample of 10 tools chosen at random, exactly 2 will be defective is

(A) 0.19
(B) 0.16
(C) 0.15
(D) 0.17

128. The density function of a random variable x is given by $f(X) = \frac{1}{2^x}$, $0 < x < 2$.

The expected value of x is

(A) $\frac{3}{4}$

(B) $\frac{4}{5}$

(C) $\frac{4}{3}$

(D) $\frac{5}{3}$

129. If $f : A \to B$ and $g : B \to C$ be functions such that the $g \circ f : A \to C$ is surjective, then the mapping g is

(A) injective
(B) surjective
(C) bijective
(D) not surjective

130. If a, b are elements of a group G such that $O(a) = 3$ and $aba^{-1} = b^2$, then $O(b)$ is

(A) 4
(B) 5
(C) 6
(D) 7

131. Let R be a ring such that $a^2 = a$ for all $a \in R$. Then, for any element b in R, $2b$ equals

(A) 0
(B) 1
(C) 2
(D) 4

132. A basis of R^3 which contains (1, 2, 0) and (1, 3, 1) is

(A) {(1, 2, 0), (1, 3, 1), (0, 1, 0), (1, 1, 0)}
(B) {(1, 2, 0), (0, 1, 0), (1, 3, 1)}
(C) {(1, 2, 1), (0, 1, 1), (1, 3, 1)}
(D) {(1, 2, 0), (1, 3, 1), (1, 1, 1), (−1, 1, 0)}

133. All the characteristics roots of a Hermitian matrix are

(A) real
(B) imaginary
(C) rational
(D) irrational

134. If $x+y+z=1$, then the least value $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$

(A) 9
(B) 8
(C) 6
(D) 4

135. If α is special root of the equation $x^8-1=0$, then $1+3\alpha+5\alpha^2+...+15\alpha^7=$

(A) $\frac{15}{\alpha-1}$

(B) $\frac{16}{\alpha-1}$

(C) $\frac{17}{\alpha-1}$

(D) $\frac{18}{\alpha-1}$

136. If $U=L\{(1,2,1),(2,1,3)\}$ and $W=L\{(1,0,0),(0,1,0)\}$ are the subspaces of R^3, then $\dim(U+W)=$

(A) 1
(B) 2
(C) 4
(D) 3

137. If $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, then

(A) $\Delta = (a-b)(b-c)(c-a)(ab-bc-ca)$
(B) $\Delta = (a-b)(b-c)(c-a)(ab+bc+ca)$
(C) $\Delta = (a+b)(b+c)(c-a)(ab+bc+ca)$
(D) $\Delta = (a+b)(b+c)(c+a)(ab+bc+ca)$

138. The direction cosines of the line joining the points $A(-4,9,6)$ and $B(-1,6,6)$ are

(A) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$

(B) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

(C) $1, -\frac{1}{\sqrt{2}}, 0$

(D) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

139. The equation of a sphere described on the line joining the points (2, –1, 4) and (–2, 2, –2) as diameter, is

(A) $x^2 + y^2 + z^2 + y + 2z - 14 = 0$

(B) $x^2 + y^2 + z^2 - y - 2z - 14 = 0$

(C) $x^2 + y^2 + z^2 + y - 2z + 14 = 0$

(D) $x^2 + y^2 + z^2 + y + 2z + 14 = 0$

140. The n^{th} order derivative of $\sin(ax+b)$ is

(A) $a^n \sin\left(ax+b+\frac{\pi}{2}\right)$

(B) $a^{n-1} \sin\left(ax+b+\frac{\pi}{2}\right)$

(C) $a^n \sin\left(ax+b+\frac{n\pi}{2}\right)$

(D) $a^{n-1} \sin\left(ax+b+\frac{n\pi}{2}\right)$

141. The value of $\int_0^a \frac{x^7}{\sqrt{\left(a^2-x^2\right)}}dx$ is equal to

(A) $\frac{16}{35a^7}$

(B) $\frac{17}{35a^7}$

(C) $\frac{19}{35a^7}$

(D) $\frac{13}{35a^7}$

142. A problem in mechanics is given to 3 students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Then the probability that the problem will be solved is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{3}{4}$

143. The vectors $X_1 = (1,1,-1,1)$, $X_2 = (1,-1,2,-1)$, $X_3 = (3,1,0,1)$

(A) are linearly dependent
(B) are linearly independent
(C) form a basis for $\mathbb{R}^3$
(D) form a basis for $\mathbb{R}^4$

144. One of the particular integral of the PDE $r - 2s + t = \cos(2x+3y)$ is

(A) $\sin(2x+3y)$
(B) $\cos(3x+2y)$
(C) $\sin(2x-3y)$
(D) $\cos(x+y)$

145. The order x^7 in a group of order 56 is

(A) 8
(B) 16
(C) 24
(D) 56

146. The characteristic of $\left(\mathbb{Z}_{189},+,\cdot\right)$ is

(A) 1
(B) 17
(C) 189
(D) ∞

147. The number of group homomorphisms from S_3 to $\mathbb{Z}_6$ is

(A) 1
(B) 2
(C) 3
(D) 4

148. The automorphism group of the group $\left(\mathbb{Z}_{10},+_{10}\right)$ is a

(A) group of order 3
(B) cyclic group of order 4
(C) non-abelian group
(D) non cyclic group of order 4

149. Let $f : A \to A$ and $B \subset A$. Then which of the following is always true.

(A) $B \subset f^{-1}(f(B))$

(B) $B = f^{-1}(f(B))$

(C) $B \subset f\left(f^{-1}(B)\right)$

(D) $B = f\left(f^{-1}(B)\right)$

150. If $\overline{A}$ and $\overline{B}$ are irrotational vectors, then $\left(\overline{A} \times \overline{B}\right)$ is

(A) irrotational
(B) rotational
(C) solenoidal
(D) flux

FINAL ANSWER KEY

Subject Name: 612 MATHEMATICS

SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key
1	C	31	C	61	A	91	B	121	A
2	C	32	B	62	C	92	C	122	D
3	D	33	A	63	D	93	B	123	D
4	B	34	B	64	C	94	A	124	D
5	D	35	B	65	A	95	D	125	C
6	C	36	C	66	A	96	B	126	B
7	A	37	B	67	C	97	D	127	A
8	C	38	B	68	C	98	B	128	C
9	A	39	A	69	C	99	A	129	B
10	D	40	B	70	C	100	B	130	D
11	D	41	C	71	B	101	A	131	A
12	D	42	A	72	B	102	C	132	B
13	C	43	D	73	A	103	D	133	A
14	C	44	B	74	C	104	A	134	A
15	C	45	C	75	B	105	C	135	B
16	C	46	C	76	B	106	C	136	D
17	C	47	D	77	A	107	B	137	B
18	D	48	C	78	B	108	C	138	A
19	D	49	A	79	D	109	A	139	B
20	B	50	A	80	B	110	A	140	C
21	D	51	D	81	C	111	C	141	A
22	B	52	C	82	C	112	D	142	D
23	A	53	A	83	B	113	C	143	A
24	C	54	B	84	C	114	D	144	A
25	B	55	B	85	A	115	A	145	A
26	C	56	A	86	B	116	B	146	B
27	C	57	D	87	A	117	D	147	C
28	C	58	B	88	A	118	B	148	B
29	D	59	B	89	C	119	C	149	A
30	B	60	B	90	A	120	B	150	C